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Couplings of a Light Dilaton and Violations of the Equivalence Principle

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Abstract

Experimental discovery of the dilaton would provide strong evidence for string theory. A light dilaton could show up in current tests of the inverse square law for gravity at sub-millimeter distances. In the large extra dimension scenario, Kaluza-Klein excitations of the dilaton can also contribute to the cooling of supernovae. In order to quantify these effects we compute the couplings of a low energy dilaton to matter. These predominantly arise from the fundamental dilaton coupling to the gluon field strength, which receives a sizable enhancement from QCD scaling. We show that detection of the dilaton will give a direct measurement of the QCD coupling constant at the string scale. Particular attention is paid to the size of equivalence principle violating dilaton couplings.

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1 Introduction

String theory predicts the existence of a scalar particle called the dilaton, with prescribed tree-level couplings at the string scale. Its discovery would be a major step toward experimentally validating string theory. It is believed that nonperturbative effects could generate a potential for the dilaton, but the mechanism and the form of this potential are presently unknown. In particular, there is no theory for the mass of the dilaton. Assuming dilaton mass generation is associated with supersymmetry breaking, a naive estimate would suggest that the dilaton mass is roughly $m \sim \Lambda_{SUSY}^2/M_{PL}$, where Λ_{SUSY} is the supersymmetry breaking scale, and M_{PL} is the Planck mass. Λ_{SUSY} could be as low as a few TeV, corresponding to a dilaton Compton wavelength on the order of a millimeter. Such a dilaton will mediate a measurable long range Yukawa force. Currently there are several experiments testing for deviations from Newton's law of gravity at sub-millimeter distances [1, 2]. In order to know what mass range for the dilaton they are sensitive to, it is necessary to know the dilaton's coupling to matter. In the large extra dimension scenarios, the dilaton can affect not only experimental tests of gravity, but also cooling rates of supernovae. In this note, we compute the leading contributions to the light dilaton coupling to matter. Our analysis and results differ from earlier calculations of the dilaton coupling [3, 4].

2 Low Energy dilaton coupling to quarks and gluons

We begin by computing the dilaton couplings assuming closed string theory above the string scale Λ_s, and an effective field theory below Λ_s consisting of the dilaton and QCD with the three light flavors of quarks (u, d, s) and an arbitrary number of heavy quarks, of which the charm quark is the lightest. As we will discuss below, to leading order the only way that high energy physics enters the calculation (such as electroweak physics, supersymmetry, grand unification) is through the value of the strong coupling, $\alpha_3(\Lambda_s)$. Therefore the results and conclusions we derive in this section are actually independent of the particle content between the weak scale and the string scale.

Assuming that the dilaton ϕ is the only light modulus, and that perturbation theory is valid at the string scale Λ_s, the lagrangian for this system at Λ_s is

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(1 - \sqrt{2}\kappa\phi)\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_i \left[\bar{q}_i i\not{D} q_i - \left(1 + \frac{\kappa}{\sqrt{2}}\phi\right) m_i \bar{q}_i q_i\right] \tag{1}$$

where $V(\phi)$ is the unknown potential for the dilaton (with minimum at $\phi = 0$), $G_{\mu\nu}$ is the gluon field strength, and

$$\kappa = \sqrt{8\pi G_N} = \frac{\sqrt{8\pi}}{M_{PL}} = [2.43 \times 10^{18}\,\mathrm{GeV}]^{-1}\ . \tag{2}$$

In eq. (1) we have only kept those terms linear in κ. In order to use this lagrangian to compute the coupling of a light dilaton to nucleons, we need to scale it down to the QCD

scale ~ 1 GeV and match onto an effective theory of hadrons. To do so, we employ the powerful techniques developed by Shifman, Vainshtein and Zakharov in ref. [5] to discuss the coupling of a light Higgs boson to nucleons (see also [6, 7, 8]). We make use of the fact that both of the operators $m_i\bar{q}_iq_i$ and $\partial_\mu S^\mu$ are renormalization group invariants, where $\partial_\mu S^\mu$ is the divergence of the scale current:

$$\partial_\mu S^\mu = \frac{\beta_3}{g_3}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + (1-\gamma_m)\sum_i m_i\bar{q}_iq_i \ , \tag{3}$$

β_3 and γ_m being the beta function for the QCD coupling g_3 and the mass anomalous dimension respectively. The coupling of the dilaton at the string scale may be rewritten in terms of these operators as

$$\mathcal{L}_\phi = \phi\frac{\kappa}{\sqrt{2}}\frac{g_3(\Lambda_s)}{\beta_3(\Lambda_s)}\left[\partial_\mu S^\mu - c\sum_i(m_i\bar{q}_iq_i)\right] \tag{4}$$

where

$$c = \left[1-\gamma_m+\frac{\beta_3}{g_3}\right]_{\mu=\Lambda_s} \ . \tag{5}$$

Note that $g_3(\Lambda_s)$ denotes the running QCD coupling evaluated at the string scale.

To compute the coupling of the dilaton to matter, we need to first integrate out the heavy quarks. This is easily done to leading order in α_3 by equating the divergence of the scale current in the theory renormalized just above the heaviest quark threshold, to the same quantity computed in the effective theory just below the charm quark mass. One finds [5]

$$\sum_{\text{heavy}} m_i\bar{q}_iq_i = \left(1-\frac{b_0^s}{b_0^\ell}\right)\left(\partial_\mu S^\mu - \sum_{\text{light}} m_i\bar{q}_iq_i\right) + \mathcal{O}\left(\alpha(m_c)\right) + \mathcal{O}\left(\alpha(m_c)\Lambda_{QCD}^2/m_c^2\right) \ . \tag{6}$$

where the QCD β-function is given by $\beta = -b_0g^3/16\pi^2 + \mathcal{O}(g^5)$, where $b_0 = (11-2/3N_f)$ and N_f is the number of quark flavors lighter than the renormalization scale. The quantity b_0^s is computed at the string scale, including the heavy quark fields, while b_0^ℓ is computed in the effective theory with only the three light quarks. Putting this altogether, the dilaton coupling may be written as

$$\mathcal{L}_\phi = -\phi\frac{K\kappa}{\sqrt{2}}\left[\partial_\mu S^\mu - \sum_{u,d,s}(m_i\bar{q}_iq_i)\right] + \dots \ , \qquad K = \frac{4\pi/b_0^\ell}{\alpha_3(\Lambda_s)} = \frac{4\pi/9}{\alpha_3(\Lambda_s)} \tag{7}$$

where the ellipses refer to the $O(\alpha_3(m_c))$ and higher order corrections [1]. We have dropped

[1] This formula disagrees with two prior calculations in the literature. Our answer is similar to the formula of Ellis et al. [4], but differs in two ways: First, eq. (40) in that paper shows the enhancement factor K proportional to $1/b_0^s$ instead of $1/b_0^\ell$ as in eq. (7) above. The difference arises from our inclusion of the effects of the heavy colored matter fields. Second, we find that the terms responsible for violations of the principle of equivalence receive the $K = \mathcal{O}(1/\alpha_3(\Lambda_s))$ enhancement, while they are found to be $\mathcal{O}(1)$ in [4]. The earlier work of Taylor and Veneziano [3] also found a logarithmic enhancement of the dilaton coupling, but otherwise differs from our result.

the terms in eq. (5) involving γ_m and β_3/g_3 evaluated at $\mu = \Lambda_s$, which are subleading in $g_3(\Lambda_s)$, the strong coupling at the string scale.

The result eq. (7) is valid at leading order in α_3 provided the dilaton is the only light modulus (if there are several light moduli then they can mix with each other). Our result eq. (7) is independent of assumptions about the nature of physics between the weak scale and the string scale, other than that the gluon coupling remain perturbative in this regime [2]. Using the measured value of $\alpha_3(M_Z)$, the coupling K can be computed in any particular model. For example, in the standard model, $1/\alpha_3(M_{PL}) \simeq 52$ and if $\Lambda_s = M_{PL}$ the enhancement factor is

$$K \simeq 73 \qquad (\text{Standard Model}, \Lambda_s = M_{PL}) \ . \tag{8}$$

In theories with additional heavy colored particles, or a lower string scale Λ_s, the dilaton coupling will be weaker, because the QCD coupling will be larger at the string scale. For example in the minimal supersymmetric extension of the standard model (MSSM) the scalar partners of the quarks and the gluinos contribute to the beta function above the weak scale, increasing the value of the strong coupling at the string scale. In this case $1/\alpha_3(M_{PL}) = 27$ and so $\Lambda_s = M_{PL}$ implies an enhancement factor

$$K \simeq 38 \qquad (\text{MSSM}, \Lambda_s = M_{PL}) \ . \tag{9}$$

It is possible to have K as small as $\mathcal{O}(1)$ if there is a large number of heavy colored states below the string scale, as occurs in many supersymmetric GUT models with $\Lambda_s = M_{PL}$. In any case, detecting the dilaton will give a direct measurement of the strong coupling constant at the string scale.

We conclude this section with an aside about the relation between the dilaton mass and its coupling, imposed by naturalness. Naturalness is simply the statement that the physical dilaton mass should be at least as large as the size of radiative corrections to the mass computed in perturbation theory. Radiative corrections to the dilaton mass due to gluon loops are quadratically divergent, and therefore sensitive to (unknown) high energy physics. However the computable standard model contribution due to gluon loops ought to serve as a lower bound on the radiative corrections. We estimate the standard model contribution with a cutoff of $\Lambda_{SM} = 1\ TeV$ to be

$$\begin{aligned} m_\phi \gtrsim [\delta m_\phi]_{SM} &\simeq \frac{1}{4\pi}\frac{\kappa}{\sqrt{2}}\frac{\alpha_3(\Lambda_{SM})}{\alpha_3(\Lambda_s)}\Lambda_{SM}^2 \\ &= 2K \times 10^{-6}\,\text{eV} \ . \end{aligned} \tag{10}$$

This lower bound is sufficiently weak that it does not constrain the unexplored parameter space accessible to experiments attempting to detect new long range forces. However, if the

[2] We explain in appendix B why the inclusion of electromagnetism does not change the form of eq. (7) to the order we are working.

scale of new physics is higher, such as $\Lambda_{SM} \sim 10\,\text{TeV}$, the naturalness constraint becomes significant.

In any case, naturalness arguments are not conclusive as one can imagine that the unnaturally small value of the cosmological constant is somehow related to similarly unnatural properties of the dilaton. In fact, the currently favored value of $\Omega_\Lambda \simeq 0.7$ implies an energy density $\sim (10^{-3}\,\text{eV})^4$, making the exploration of (sub)-millimeter gravity especially intriguing [9, 10, 11].

3 The dilaton coupling to nucleons

To the order we are working, the effective nucleon-dilaton coupling is given by

$$\mathcal{L}_{N\phi} = \phi\,(y_p \bar{p} p + y_n \bar{n} n) \ , \tag{11}$$

where the Yukawa couplings are given by the matrix element

$$y_i = -\frac{\kappa}{\sqrt{2}} K \langle N_i | \partial_\mu S^\mu - \chi | N_i \rangle \ , \qquad \chi \equiv (m_u\, \bar{u} u + m_d\, \bar{d} d + m_s\, \bar{s} s) \ , \tag{12}$$

where N_i is the nucleon doublet. As the matrix element of the divergence of the scale current is

$$\langle N_i | \partial_\mu S^\mu | N_i \rangle = M_i \ , \tag{13}$$

the Yukawa potential between two nucleons of types i, j may be written as

$$V_{ij}(r) = -\frac{y_i y_j}{4\pi r} e^{-M_\phi r} = -K^2\, \frac{G_N \mathcal{M}_i \mathcal{M}_j}{r} e^{-M_\phi r} \tag{14}$$

where $K = (4\pi/9)/\alpha_3(\Lambda_s)$ is the enhancement factor in eq. (7),

$$\begin{aligned} \mathcal{M}_p &= M_p\,(1 - \hat{\chi}_+ - \hat{\chi}_-) \ , \\ \mathcal{M}_n &= M_n\,(1 - \hat{\chi}_+ + \hat{\chi}_-) \ , \\ \hat{\chi}_\pm &= \frac{1}{2}\left(\frac{\langle p|\chi|p\rangle}{M_p} \pm \frac{\langle n|\chi|n\rangle}{M_n} \right) \ . \end{aligned} \tag{15}$$

We compute $\chi_\pm$ to leading order in chiral perturbation theory in appendix A, finding

$$\begin{aligned} \hat{\chi}_+ &= (2.7 \pm 0.8) \times 10^{-1} \ , \\ \hat{\chi}_- &= -(1.5 \pm 0.5) \times 10^{-3} \ , \end{aligned} \tag{16}$$

where the errors quoted reflect an estimated $\sim 30\%$ violation of $SU(3)$ symmetry.

In the isospin symmetric limit $\hat{\chi}_- = 0$ we see that the dilaton mediated force between nucleons obeys the principle of equivalence and is $K^2(1-\hat{\chi}_+)^2$ times stronger than gravity; the factor K was computed in eq. (7), while $(1-\hat{\chi}_+)^2 \simeq 0.5$.

The isospin violating term proportional to $\hat{\chi}_-$ contributes to a force which violates the principle of equivalence. The difference between the accelerations of a proton and a neutron in a dilaton field is proportional to

$$\Delta_{p,n} = \left(\frac{\mathcal{M}_p}{M_p} - \frac{\mathcal{M}_n}{M_n}\right) = -2\hat{\chi}_- \ , \tag{17}$$

a $\sim 0.3\%$ effect relative to the common acceleration.

One might suppose that the inclusion of electromagnetism would give rise to comparable equivalence principle violating effects. However, the leading electromagnetic effects are already accounted for in eq. (15) when the physical nucleon masses are used; additional electromagnetic contributions are suppressed by an additional power of α_{em} or $1/K$, as discussed in appendix B.

4 The dilaton coupling to atoms

The Yukawa potential between two atoms i and j, from dilaton exchange, is

$$V_{ij}(r) = -K^2\,\frac{G_N \mathcal{M}_i \mathcal{M}_j}{r} e^{-M_\phi r} \tag{18}$$

where $K = (4\pi/9)/\alpha_3(\Lambda_s)$ is the enhancement factor in eq. (7) and

$$\begin{aligned} \mathcal{M}_i &= \langle i|\partial_\mu S^\mu - \chi - m_e \bar{e}e|i\rangle \\ &= M_i - \langle i|\chi + m_e \bar{e}e|i\rangle \ , \end{aligned} \tag{19}$$

where M_i is the mass of atom i, and we have included the electron coupling of the dilaton. The mass of an atom with charge Z and neutron number N may be written as

$$M = Z\,M_p + Z\,m_e + N\,M_n - \mathcal{E} \ , \tag{20}$$

where $\mathcal{E}$ is the binding energy, $\mathcal{E} \lesssim A \times (9\,\mathrm{MeV}\,)$, where $A = (Z+N)$ is the atomic number. Thus we can write the matrix element in eq. (19) as

$$\begin{aligned} \langle i|\chi + m_e \bar{e}e|i\rangle &= \sum_{a=u,d,s,e} m_a\, dM_i/dm_a \\ &= (Z_i M_p + N_i M_n)\,\hat{\chi}_+ + (Z_i M_p - N_i M_n)\,\hat{\chi}_- + Z_i m_e - \sum_a m_a \frac{d\mathcal{E}}{dm_a} \\ &\simeq M_i \left[\hat{\chi}_+ + \left(\frac{Z_i - N_i}{A_i}\right)\hat{\chi}_- + \frac{m_e}{M_N}\frac{Z_i}{A_i} - \sum_a m_a \frac{d(\mathcal{E}/M_i)}{dm_a}\right] \end{aligned} \tag{21}$$

	$\left(\frac{Z_i - N_i}{A_i}\right)\hat{\chi}_-$	$\frac{m_e}{M_N}\frac{Z_i}{A_i}$	$\frac{\mathcal{E}}{M_i}\hat{\chi}_+$
${}^{9}_{4}$Be	1.7	2.42	19
${}^{27}_{13}$Al	0.56	2.62	24
${}^{28}_{14}$Si	0	2.72	24
${}^{63}_{29}$Cu	1.2	2.51	25
${}^{208}_{82}$Pb	3.2	2.15	23

Table 1. *The last three terms in eq. (23), times* 10^4*, evaluated for various common isotopes; we have used the central values for* $\hat{\chi}_\pm$ *from first order chiral perturbation theory, eq. (16). The three terms are all small — the first due to isospin violation, the second due to the small electron mass, and the third due to the smallness of nuclear binding energy. Note that only the differences between entries for different isotopes, entering* Δ_{ij} *in eq. (24) are experimentally relevant for violation of the equivalence principle.*

In the last line we expanded to linear order in $\hat{\chi}_-$, m_e/M_i, $(M_n - M_p)/M_N$ and $\mathcal{E}/M_i$; we have retained terms of order $(\mathcal{E}/M_i)\hat{\chi}_+$, which are of comparable magnitude to $Z_i m_e/M_i$ and $\hat{\chi}_-$.

We do not know how to compute the last term in eq. (21), the dependence of the binding energy on the quark and elecron masses. It is expected to be dominated by the $m_s\bar{s}s$ operator, and on dimensional grounds it should be of comparable magnitude to $(\mathcal{E}/M_i)\hat{\chi}_+$. We therefore define the parameters ξ_i by

$$\sum_a m_a \frac{d(\mathcal{E}/M_i)}{dm_a} \equiv \xi_i \frac{\mathcal{E}}{M_i}\hat{\chi}_+ \ , \tag{22}$$

and expect $|\xi_i| \sim 1$.

Assembling these results we find that the parameter $\mathcal{M}_i$ appearing in the force law eq. (18) is

$$\mathcal{M}_i \simeq M_i\left(1 - \hat{\chi}_+ - \left(\frac{Z_i - N_i}{A_i}\right)\hat{\chi}_- - \frac{m_e}{M_N}\frac{Z_i}{A_i} - \xi_i\frac{\mathcal{E}}{M_i}\hat{\chi}_+\right) \ . \tag{23}$$

As for nucleons, we find that the equivalence principle conserving part of the force is $\sim K^2(1-\hat{\chi}_+)^2 \sim 0.5K^2$ times the gravitational force. As discussed above, the numerical value of K^2 is unknown, and could be $\mathcal{O}(1) - \mathcal{O}(10^3)$; it is directly related to the gluon coupling at the string scale through eq. (7).

Violations of the equivalence principle in the force between atoms i and j are proportional to

$$\Delta_{i,j} = \left(\frac{\mathcal{M}_i}{M_i} - \frac{\mathcal{M}_j}{M_j} \right) \tag{24}$$

which gets contributions from the last three terms in eq. (23). In Table 1 we give the sizes for these equivalence principle violating terms for several isotopes, given the chiral perturbation theory calculation of $\hat{\chi}_+$ in eq. (16). Note that by choosing to compare different pairs materials, it is in principle possible to have the equivalence principle violating parameter Δ_{ij} be selectively sensitive to each of the last three terms in eq. (23).

5 Dilaton couplings with large extra dimensions

String theory has additional spatial dimensions beyond the three we have observed. In writing the Lagrangian in eq. (1) we have assumed that these extra dimensions are small (size of order $1/\Lambda_s$) and that only the particular combination of higher dimensional metric components and higher dimensional dilaton field that appears as a factor scaling the effective four dimensional action remains light. It is this combination that is the effective four dimensional dilaton field ϕ that occurs in eq. (1). Recently the possibility of having the string scale of order the weak scale has been explored [12, 13, 14, 15]. The large effective four dimensional Planck mass arises because some of the additional extra dimensions have sizes much larger than $1/\Lambda_s$. In these scenerios enhancement of the low energy dilaton coupling to matter is different from that presented in eq. (7) [16]. This difference arises for several reasons. Firstly to avoid experimental constraints the matter fields must be confined to a three-brane and at leading order in string perturbation theory the effective field theory arises from string amplitudes on the disk instead of the sphere. It is possible for there to be a dilaton mass term on the brane as well as in the bulk. Even with toroidal compactification and no warp factor a brane mass term gives rise to Kaluza-Klein excitations of the dilaton that are not just plane waves and a lowest mode that is not constant in the extra dimensions. This can dramatically influence the dilaton couplings to matter on the brane. In this section we assume that the dilaton couplings have the form one would deduce in the simplest possible case, where its lowest mass mode is constant in the extra dimensions and its Kaluza-Klein excitations are simply plane waves.

With n large extra dimensions the low energy dilaton couplings to quarks and gluons are given by eq. (7) but the enhancement factor is changed to

$$K_n = \frac{\sqrt{4+2n}}{4} K = \frac{\pi\sqrt{4+2n}}{9\ \alpha_3(\Lambda_s)}. \tag{25}$$

Note that K_n is a factor of $\sqrt{4+2n}/2$ larger than K from the normalization appropriate to rescaling the $4+n$ dimensional metric to go from the string metric to the Einstein metric, and an additional factor of 2 smaller because the couplings arise from string amplitudes

calculated on the disk. With two large extra dimensions and the string scale equal to 10 TeV, the enhancement factor in eq. (25) is $K_2 \simeq 15$. There is one more complication that effects the calculation of the force between matter from dilaton exchange. The dilaton may not be a mass eigenstate, it might mix with components of the higher dimensional metric. Assuming that this mixing angle θ is not small [3] , the force between matter from exchange of the lowest mass eigenstate is then given by eq. (14) (for nucleons) or eq. (18) (for atoms) but with K replaced by K_n and a factor of $\cos^2\theta$ inserted.

For two large extra dimensions an important constraint on the size of the extra dimensions comes from the rate of supernova cooling associated with emission of Kaluza Klein excitations of the graviton and of a combination components of the higher dimensional metric which is also sometimes called the dilaton. If the string theory dilaton is also light then it will contribute to this constraint. We can estimate the size of this effect using the recent results of Hanhart *et al.* [17]. The emissivity due to Kaluza-Klein excitations of the string theory dilaton is larger than ϕ emission computed in ref. [17] by a factor of

$$f = \frac{3(n+2)K_n^2}{4}. \tag{26}$$

For $n = 2$ and $\Lambda_s \simeq 10$ TeV this enhancement is $f \simeq 550$. This results in a total emissivity approximately 8 times larger than computed in ref. [17] for a given size R of the two flat extra dimensions. The upper bound on R becomes stronger by $\sqrt{8}$, or $R < 2.5 \times 10^{-4}$ mm.

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[3]For small θ terms that we neglected, which are not enhanced by $1/K_n$, may be important.

A Computing $\mathcal{M}_{p,n}$ in leading order chiral perturbation theory

The matrix elements $\hat{\chi}_\pm$ may be easily computed at leading order in chiral perturbation theory, using the $SU(3)_L \times SU(3)_R$ baryon chiral lagrangian. Using the notation of ref. [18], the mass terms in this Lagrangian are,

$$\mathcal{L}_m = -m_0 \mathrm{Tr}\, \bar{B}B + 2a_1 \mathrm{Tr}\, \bar{B}MB + 2a_2 \mathrm{Tr}\, \bar{B}BM + 2a_3 \mathrm{Tr}\, M \mathrm{Tr}\, \bar{B}B \ , \tag{27}$$

where M is the quark mass matrix $\mathrm{diag}\{m_u, m_d, m_s\}$, and B is the baryon octet matrix. The constants m_0 and a_i times the quark masses can be determined in terms of the octet baryon masses, the quark mass ratios determined in the meson sector, and the pion-nucleon "Σ-term",

$$\Sigma_{\pi N} \equiv \left(\frac{m_u + m_d}{2}\right) \langle N|\bar{u}u + \bar{d}d|N\rangle \tag{28}$$

extracted from $\pi - N$ scattering data. The leading order result (i.e, to linear order in quark masses) is

$$\begin{aligned}
\hat{\chi}_+ &= \tfrac{1}{M_N}\langle N|m_s\bar{s}s + \left(\tfrac{m_u+m_d}{2}\right)(\bar{u}u + \bar{d}d)|N\rangle \\
&= \tfrac{1}{M_N}\left[\Sigma_{\pi N}\left(1 + \tfrac{m_s}{m_u+m_d}\right) - (M_\Xi + M_\Sigma - 2M_N)\left(\tfrac{m_s}{2m_s-m_u-m_d}\right)\right] \ , \\
\hat{\chi}_- &= \tfrac{1}{M_N}\langle p|\left(\tfrac{m_u-m_d}{2}\right)(\bar{u}u - \bar{d}d)|p\rangle \\
&= \tfrac{1}{M_N}\left[-(M_\Xi - M_\Sigma)\left(\tfrac{m_d-m_u}{2m_s-m_u-m_d}\right)\right]
\end{aligned} \tag{29}$$

The quark mass ratios have been determined from the measured pseudoscalar octet meson masses to be [19]

$$\frac{m_s}{m_u} = 34.4 \pm 3.7 \ , \qquad \frac{m_s}{m_d} = 18.9 \pm 0.8 \tag{30}$$

while recent analyses of $\pi - N$ scattering yields [20, 21, 22]

$$\Sigma_{\pi N} \simeq 45\,\mathrm{MeV}. \tag{31}$$

These numbers, along with the measured hyperon masses yield

$$\begin{aligned}
\hat{\chi}_+ &= \tfrac{1}{2}\left(\tfrac{\mathcal{M}_p}{M_p} + \tfrac{\mathcal{M}_n}{M_n}\right) = (2.7 \pm 0.8) \times 10^{-1} \ , \\
\hat{\chi}_- &= \tfrac{1}{2}\left(\tfrac{\mathcal{M}_p}{M_p} - \tfrac{\mathcal{M}_n}{M_n}\right) = -(1.5 \pm 0.5) \times 10^{-3} \ ,
\end{aligned} \tag{32}$$

where we have assigned (somewhat arbitrarily) a 30% error estimate typical for $SU(3)$ violation. Note that in the standard model with $\Lambda_s = M_{PL}$, $K \simeq 73$ and the dilaton force between nucleons is approximately 2.6×10^3 times stronger the gravity, while the component violating the principle of equivalence is approximately 8 times stronger than gravity [4].

B Electromagnetic effects on dilaton coupling to nucleons

The above analysis is apparently incomplete, as it neglects electromagnetic effects. One might think that the inclusion of electromagnetism would change the equivalence violating potential by an amount comparable to that due to the $(m_u - m_d)$ mass difference, as the two sources of isospin violation contribute comparably to the $n - p$ mass splitting. While true, such effects are already accounted for when the physical proton and neutron masses (which include electromagnetic contributions) are used in eq. (15). All additional electromagnetic contributions to equivalence principle violation are relatively suppressed by either a factor of $1/K$ or α_{em}.

Including electromagnetic interactions gives rise to a new coupling of the dilaton at the string scale. In addition to the terms in eq. (1) there is now a term

$$\mathcal{L}_\phi^{(em)} = -\frac{1}{4}(1 - \sqrt{2}\kappa\phi) F_{\mu\nu} F^{\mu\nu}. \tag{33}$$

However when the dilaton couplings are scaled down from the string scale its coupling to the electromagnetic field strength tensor, shown above, does not get enhanced by the large factor K and hence we can neglect the dilaton coupling in eq. (33). The divergence of the scale current has a piece proportional to the the square of the electromagnetic field strength tensor and so the low energy dilaton coupling is changed from that in eq. (7) to

$$\mathcal{L}_\phi = \phi \frac{\kappa}{\sqrt{2}} K \left[\partial_\mu S^\mu - \chi - \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu} \right]. \tag{34}$$

The last term proportional to the square of the electromagnetic field strength tensor has neutron and proton matrix elements of order α_{em}^2 and can be neglected. The dominant electromagnetic effect is the electromagnetic correction to the matrix element of the term in $\partial_\mu S^\mu$ proportional to the square of the gluon field strength tensor and it gives the contribution to the neutron and proton masses of order $\alpha_{em} \Lambda_{QCD}$. This has already been included by using the full nucleon masses in eq. (15).

[4] At leading order in chiral perturbation theory, χ_+ is dominated by the strange matrix element $\langle p | m_s \, \bar{s}s | p \rangle \simeq 218\,\mathrm{MeV}$. This value is somewhat controversial [23]. Reducing the value of $\langle p | m_s \, \bar{s}s | p \rangle$ has the effect of boosting the overall strength of the force between nucleons from dilaton exchange. For example, if $\langle p | m_s \, \bar{s}s | p \rangle = 0$, at short distance the dilaton force between nucleons is enhance to $\sim 5 \times 10^3$ stronger than gravity.

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